ATS 620
Fall 2011
Problem Set #7
Due 28 October 2011

1. (10 points) Calculate the radius and mass of an ice crystal after it has grown by deposition in a water-saturated environment at –10°C for 30 min. Let the shape of the crystal be a thin disk of constant thickness 10 μm. The capacitance C is 2r/π. The particle density is 0.1 g cm^{-3}.

$$\frac{dm}{dt} = \frac{4\pi C(S_i - 1)}{A+B} \; ; \; A = \frac{L_s}{KT}\left[\frac{L_s}{R_v T} - 1\right] \; ; \; B = \frac{R_v T}{e_i(T) D_v}$$

S_i (at water saturation and -10°_C) = 1.1021720

$T = 263.15 K$; $K_a = 0.0238696 \; J m^{-1} s^{-1} K^{-1}$ at $T = 10^\circ C$; $P = 800 mb$

$C = 2r/\pi$; $D_v = 2.48828e{-5} \; m^2/s$

$\Delta t = 1800$ sec

$H = 1e{-5} m$; $\rho_i = 100 \, Kg \, m^{-3}$

$L_s = 2833e3 \; J \, Kg^{-1}$

$$\frac{dm}{dt} = \left(3.0155644e{-8} \; Kg \, m^{-1} s^{-1}\right) r = Bob \cdot r$$

$$\frac{dm}{dt} = \frac{d}{dt}\Big(\underbrace{\pi r^2 H}_{\text{Volume of Cylinder}} \rho_i\Big) \Rightarrow \frac{dm}{dt} = \pi H \rho_i \left(2r\frac{dr}{dt}\right) = BOB \cdot r$$

$$\frac{dr}{dt} = \frac{(BOB)\cancel{r}}{2\cancel{r}\pi H \rho_i} = \frac{Bob}{2\pi H \rho_i} = \underbrace{4.7994197e{-6} \; m^{-1} s^{-1}}_{X}$$

$$\int_{r_i}^{r_F} dr = X \int_{t=0}^{t=30\,min} dt \Rightarrow \Delta r = X(1800 s) = 0.00864 \, m = 8.64 \, mm$$

Assume $r_f >> r_i$, then $\boxed{r_f = 8.64 mm}$

$m_f = \rho_i \pi r^2 H$ $\boxed{m_f = 2.344 \times 10^{-7} Kg}$

2. (10 points) Calculate the depth of cloud required for an ice crystal, which starts off as a plane plate with an effective diameter of 1 mm and a mass of 0.01 mg, to grow into a spherical graupel particle 1 mm in diameter if it falls through the cloud layer which contains a liquid water content of 0.5 g m^{-3}. Assume the collection efficiency is 0.6. Assume the density of the final graupel particle is 100 kg m^{-3} and that the terminal fall velocity of the particle (in m s^{-1}) is given by $v_t = 2.4\,M^{0.24}$, where M is the mass of the particle in milligrams. Let the updraft velocity in the cloud layer be constant with height and equal to 50 cm s^{-1}.

$$\frac{dm}{dt} = \pi r^2 V(r) W_L E_c$$ Assume V(collector) >> V(collected) ∴ V(r)

$$\frac{dm}{dt} = \frac{dm}{dh}\frac{dh}{dt} = \frac{dm}{dh}(V-w) = \pi r^2 V(r) W_L E_c$$

$$\frac{V-w}{V} dm = \pi r^2 W_L E_c\, dh$$

$$\left(1-\frac{w}{V}\right)dm = \pi r^2 W_L E_c\, dh$$

$$dm - \frac{w}{2.4\,m^{0.24}}dm = \pi r^2 W_L E_c\, dh$$

Integrate,

$$\int_{m_i}^{m_f}\left(1 - \frac{w}{2.4} m^{-0.24}\right)dm = \pi r^2 W_L E_c \int_{h_i}^{h_f} dh$$

$$\left. m - \frac{w}{(2.4)(0.76)} m^{0.76}\right]_{m_i}^{m_f} = \pi r^2 W_L E_c \Delta H$$

Plugging in values

$m_i = 1\times10^{-8}$ kg

$m_f = \frac{4}{3}\pi r^3 \rho_i$ (for sp[herical] graupe[l])

$m_f = \frac{4}{3}\pi (5e\text{-}4\,m)^3 (100\,kg\,m^{-3})$

$m_f = 5.236\,e\text{-}8$ kg

For V/M relation,

$m_i = 0.01\,m[g]$

$m_f = 0.05236$ mg

$$\Delta H = \frac{(m_f - m_i) - \frac{w}{(1.824)}\left(m_f^{0.76} - m_i^{0.76}\right)}{\pi r^2 W_L E_c}$$

$$\boxed{\Delta H = 91.2714 \text{ meters}}$$

watch units!

$$\frac{[mg] - \frac{[m\,s^{-1}]}{[m\,s^{-1}]}(mg)}{[m^2][kg\,m^{-3}]} = \underbrace{mg\,kg^{-1}}_{\text{factor of } 1e^6}\,m$$

$0.25\ mm = r_i$

3. (10 points) Compute the time required for a spherical snowflake with an initial diameter of 0.5 mm to increase in size by aggregation to a final diameter of 2 mm if it falls through a cloud containing small ice crystals present in the amount of 0.5 g m^{-3}. Assume E_c = 0.7 (collection efficiency) and the density of the snowflake is 100 kg m^{-3}. Let the difference in the fallspeeds between the snowflake and ice crystal be constant at 1 m s^{-1}. Assume that the continuous collection model applies to this situation.

$1\ mm = r_f$

$$\frac{dm}{dt} = \pi r^2 \left(V_1(r) - V_2(r_2)\right) W_i E_c$$

$V(r) = V_1 - V_2 = 1\ m\ s^{-1}$

$W_i = 5e{-4}\ kg\ m^{-3}$

$\rho_i = 100\ kg\ m^{-3}$

$E_c = 0.7$

$$\frac{dm}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3 \rho_i\right) = \pi r^2 V(r) W_i E_c$$

$$4\pi r^2 \rho_i \frac{dr}{dt} = \pi r^2 V(r) W_i E_c$$

$$\frac{dr}{dt} = \frac{V(r)\, W_i E_c}{4\rho_i}$$

$$\int_{r_i}^{r_f} dr = \frac{V\ \ E_c}{4\rho_i}\int_{T_i}^{t_f} dt$$

$$\Delta t = \frac{r_f - r_i}{\left(\frac{V\, W_i E_c}{4\rho_i}\right)} = \left(1\times10^{-3} m - 2.5\times10^{-4} m\right)\left(\frac{4\ (100\ Kg\ m^{-3})}{(1\ m\ s^{-1})(5\times10^{-4}\ Kg\ m^{-3})(0.7)}\right)$$

$$\boxed{\Delta t = 857.143\ sec = 14.2857\ min}$$

4. (20 points) Reproduce Figure 12.14 from Lamb and Verlinde using your language of choice. Show on the figure (either by hand or programming) which region of the plot you might expect the following types of deep convection to associated with:

1) Continental mid-latitude
2) Maritime tropical

Explain your reasoning for these choices.

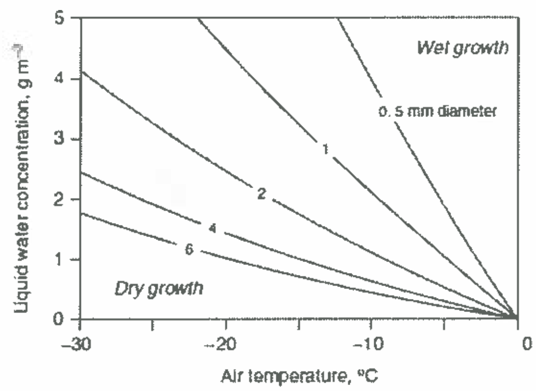
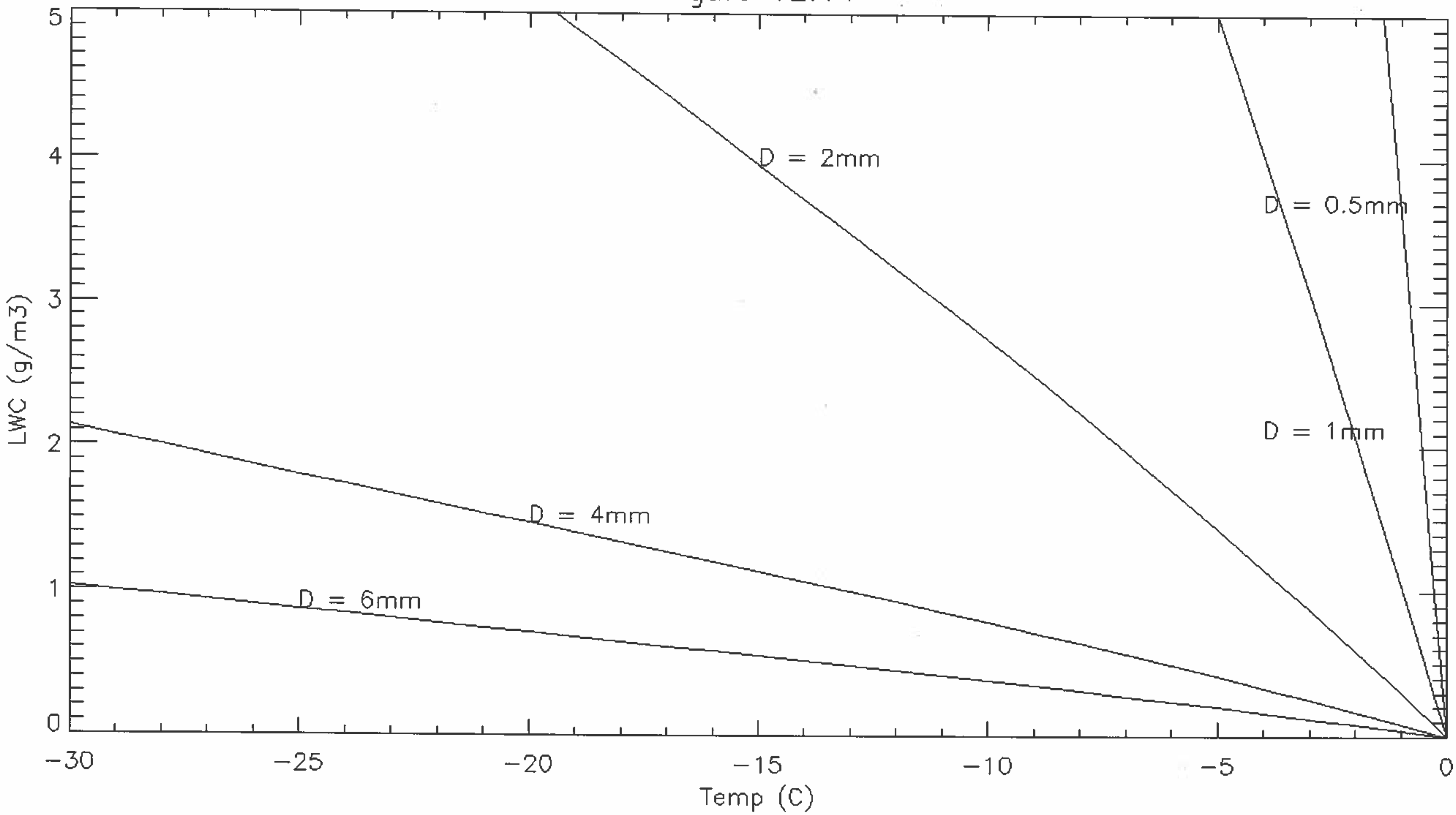


Figure 12.14 Threshold liquid water concentrations for wet growth as a function of ambient air temperature and hailstone diameter. Adapted from Young (1993).

Wet versus Dry growth is not related to Continental versus Maritim deep convection. Both air masses experience wet and dry growth. The main difference between Con. vs Mar. is cloud droplet distributions. Maritime generally has fewer, but larger liquid drops as compared with a Continental air mass. Ultimately, the LWC for both air masses are comparable. But, it could be argued that because updraft speeds are generally stronger for Continental Deep convection, they could support more liquid water.

Summary: The entire figure is for both air masses.

Figure 12.14



D = 2mm
D = 0.5mm
D = 1mm
D = 4mm
D = 6mm
LWC (g/m3)
Temp (C)
5
4
3
2
1
0
−30
−25
−20
−15
−10
−5
0