ATS 620
Fall 2011
Problem Set #8
Due 9 November 2011

1. (10 points) Determine the electric charge that would have to be separated for each collision of an ice crystal with a hailstone in a thunderstorm in order to generate charge at a rate of 1 C km^{-3} min^{-1}, where C is a Coulomb. Assume that the concentration of ice crystals is 10^5 m^{-3}, their fall speed is negligible compared to that of the hailstones, and they are collected with an efficiency of unity by the hailstones. The hailstones may be considered to be spherical with a constant radius of 2 mm and a density of 500 kg m^{-3}. Assume that the precipitation falls only as hail and that the precipitation rate is 5 cm/hr.

The rate of generating charge is related to the amount of charge per collision between a hailstone and an ice crystal and the number of collisions, assuming charging only occurs via these collisions:

$$\Rightarrow \frac{dQ}{dt} = QK \quad \text{, where } Q = \text{charge}, K = \text{total \# of collisions}$$

K depends on the concentrations, velocities, and areas of the hailstones and ice crystals, and collision efficiency, so:

$$K = \pi(r_h + r_i)^2 (V_h - V_i)\, n_h(r_h)\, n_i(r_i) \text{ ; Assume } r_h >> r_i \text{ ; } V_h >> V_i$$

$$r_h = 2mm = 2 \times 10^{-3}\, m \text{ ; } n_i = 10^5\, m^{-3}$$

To determine $V_h n_h$, consider the precip. rate (P):

$$P = \frac{V_h n_h M_h}{\rho_i} \text{ , } P = 5\, cm\, h^{-1} = 1.389 \times 10^{-5}\, m\, s^{-1} \text{, } \rho_i = 500\, kg\, m^{-3}$$

$$M_h = \rho_i \left(\tfrac{4}{3}\pi r_h^3\right) = (500\, kg\, m^{-3})\left(\tfrac{4}{3}\pi\right)(2 \times 10^{-3}\, m)^3 = 1.68 \times 10^{-5}\, kg$$

Then, $$V_h n_h = \frac{P \rho_i}{M_h} = \frac{(1.389 \times 10^{-5}\, m\, s^{-1})(500\, kg\, m^{-3})}{(1.68 \times 10^{-5}\, kg)} = 413.4\, m^{-2} s^{-1}$$

$$K = \pi r_h^2 (V_h n_h) n_i = \pi (2 \times 10^{-3}\, m)^2 (413.4\, m^{-2} s^{-1})(10^5\, m^{-3}) = 519.5\, m^{-3} s^{-1}$$

Since $\frac{dQ}{dt} = QK$, $Q = \frac{1}{K}\frac{dQ}{dt}$, where $\frac{dQ}{dt} = \frac{1\, C}{Km^3\, min} \times \frac{1\, min}{60\, sec} \times \left(\frac{1\, Km}{1000\, m}\right)^3 = 1.67 \times 10^{-11}\, C\, m^{-3} s^{-1}$

$$\therefore Q = \left(\frac{1}{519.5\, m^{-3} s^{-1}}\right)(1.67 \times 10^{-11}\, C\, m^{-3} s^{-1}) = \boxed{3.2 \times 10^{-14}\, C}$$

2. (20 points) This problem studies the conversion of gravitational power associated with falling precipitation to electrical power, the latter associated with the presence of an electric field in a thundercloud. The ratio of electrical power to gravitational power (F) can be written as

$$qEV \,/\, MgV_0 = q^*V \,/\, V_0$$

where q is electrical charge, E is the electric field, V is the fallspeed of charged particles in an electric field, M is precipitation mass and V_0 is the fallspeed in the absence of an E field. Williams and Lhermitte (1983, JGR-Atmospheres) studied this problem and found that the value of q*, a non-dimensional charge magnitude, is 0.55 when conditions are optimal for the conversion of gravitational power to electrical power.

a. With the relationship of

$$V = V_o(1 - q^*)^{0.8}$$

calculate the ratio of electrical power to gravitational power. This value sets an upper limit on the conversion of gravitational power to electrical power. Why is this process relatively inefficient?

b. Calculate the gravitational power density (W/m^3) for a precipitation rate of 1 mm/hr.

c. Using the ratio computed in (a), what is the maximum realizable amount of electrical power conversion per mm/hr of precipitation?

a) $F = \dfrac{qEV}{MgV_0} = \dfrac{q^* V}{V_0} = \dfrac{q^* V_0 (1-q^*)^{0.8}}{V_0} = q^*(1-q^*)^{0.8}$

Using $q^* = 0.55$, $F = 0.55(1-0.55)^{0.8} = \boxed{0.29 = F}$

This F represents the efficiency of gravitational power by gravitational settling of hydrometeors to electrical power in T-storms, Therefore, gravitational setting is responsible for only 29% of storm electricity b)

gravitational power density → $\rho_{GP} = P \rho_w g$ ← Precip Rate, Water density, gravity

$$[W\,m^{-3}] = \left[\frac{m}{s}\right]\left[\frac{kg}{m^3}\right]\left[\frac{m}{s^2}\right] = kg\,m^{-1}s^{-3}$$

$$\left[\frac{kg\,m^2}{s^3}\right]\left[\frac{1}{m^3}\right] = [kg\,m^{-1}s^{-3}] \quad \checkmark \text{ units work}$$

$$P = 1\,\frac{mm}{hr} \times \frac{1\,m}{1000\,mm} \times \frac{1\,hr}{3600\,sec} = 2.778\times10^{-7}\,m\,s^{-1}$$

$\rho_w = 1000\,kg\,m^{-3}$

$g = 9.81\,m\,s^{-2}$

$$\rho_{GP} = (2.778\times10^{-7}\,m\,s^{-1})(1000\,kg\,m^{-3})(9.81\,m\,s^{-2}) = \boxed{0.0027\,W\,m^{-3} = \rho_{GP}}$$

c)

From (a): $F = 0.29$, which is the upper limit of grav. power to elec. power

From (b): $\rho_{GP} = 0.0027\,W\,m^{-3}$, which is the grav. power density for a $1\,mm\,hr^{-1}$ precip rate.

Therefore, the maximum realizable amount of electrical power conversion per $mm\,hr^{-1}$ of precipitation is equal to the product of (a) and (b)

$$(0.29)(0.0027\,W\,m^{-3}) = \boxed{7.83\times10^{-4}\,W\,m^{-3}}$$

3. (20 points) Using your programming language of choice, generate a figure that relates storm intensity to lightning. Be creative and explain your reasoning and methodology for your figure.

See code for description and algorithm of plot.

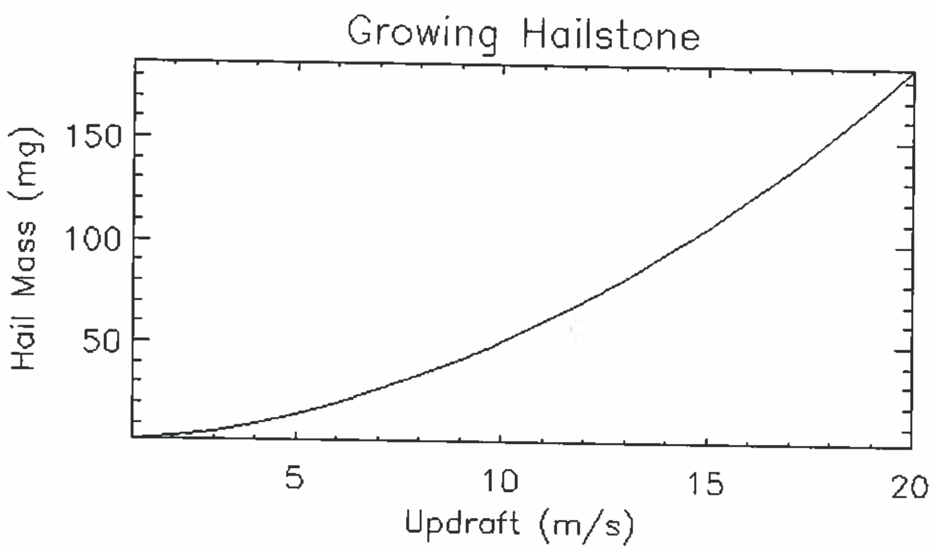
Equations used:

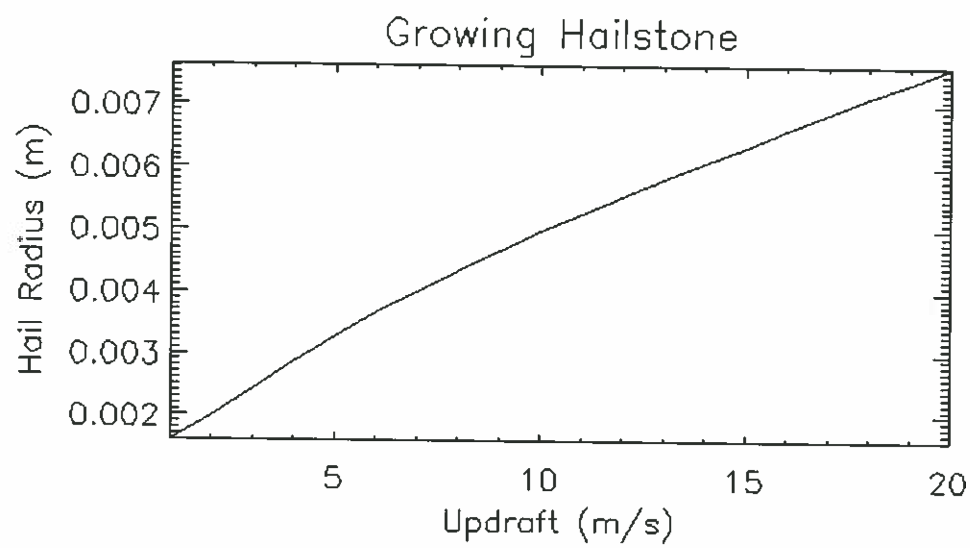
drop/updraft speed → V_d; hail fall speed → V_h

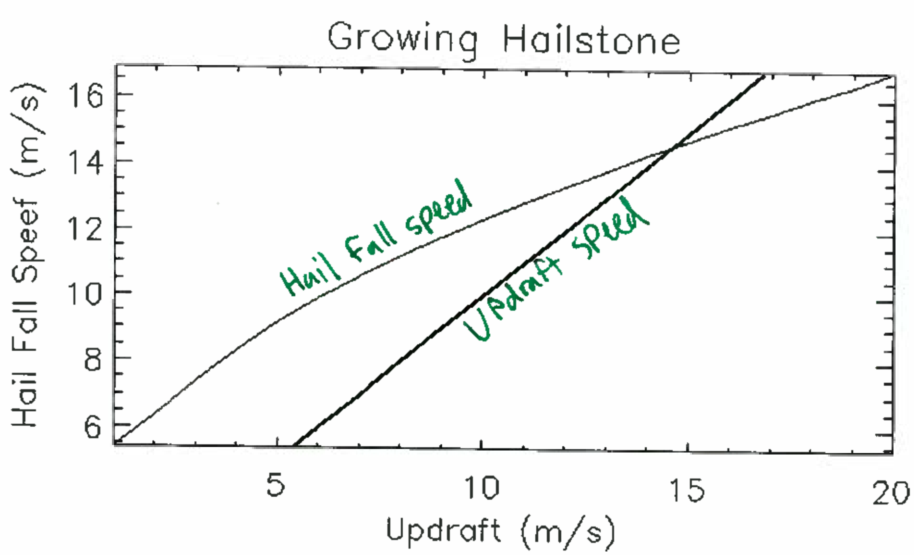
$$\frac{dm}{dt} = \pi (V_d - V_h)^2 W_L E_c$$

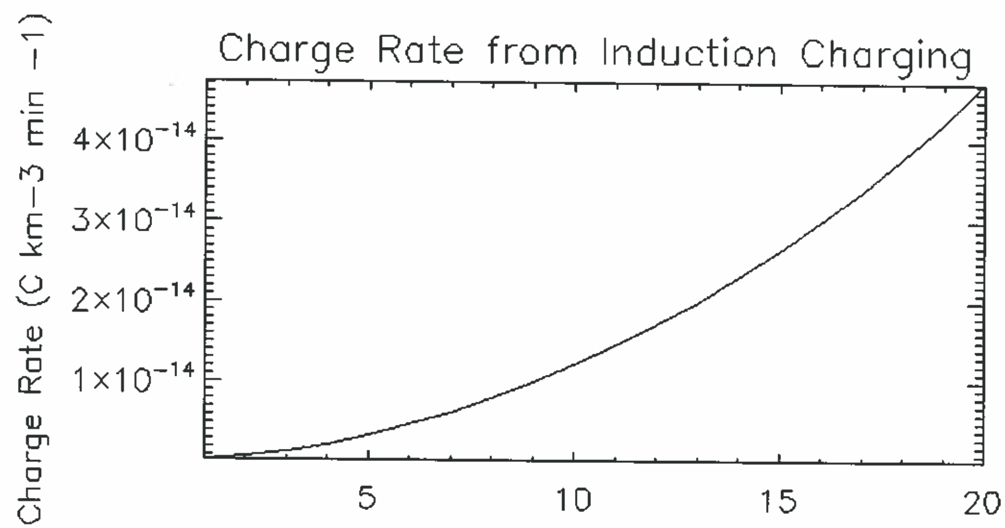
$$V_h = 4.8 M^{0.24}$$

$$\frac{dQ}{dt} = \pi (r_h + r_d)^2 E n_h n_d \Delta v \delta q \quad (14.2 \text{ L\&V})$$


Growing Hailstone
Hail Mass (mg)
150
100
50
5
10
15
20
Updraft (m/s)


Growing Hailstone
Hail Radius (m)
0.007
0.006
0.005
0.004
0.003
0.002
5
10
15
20
Updraft (m/s)


Growing Hailstone
Hail Fall Speef (m/s)
16
14
12
10
8
6
Hail Fall speed
Updraft speed
5
10
15
20
Updraft (m/s)


Charge Rate from Induction Charging
Charge Rate (C km−3 min −1)
4×10−14
3×10−14
2×10−14
1×10−14
5
10
15
20

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pro hw8

;For this question, I will run an iterative loop that adds 1 m/s to
;the updraft strength for each iteration, which I define to be 10
;seconds.  The addition of updraft speed will affect the growth rate of
;the hydrometeors (growing by riming) and subsequently affect the
;charging rate through the difference in radii and fall
;speeds. Looping over 20 iterations that add 1m/s of updraft every 10
;seconds, starting with 1 m/s, means that the hailstone encounters an
;updraft from 1 m/s to 20 m/s over 3 min 20 sec.  This is a reasonable
;amount of time for a severe storm.

;initial values and controls of loop
nit=20
dt=10.

;Assume reasonable constants
mi=1.                          ;initial ice mass (mg)
vd0=1.
nd=1.e5                        ;cloud drop concentration (m-3)
nh=1.                          ;hail concentration (m-3)
ec=0.7                         ;collection efficiency (unitless)
wl=1.e-3                       ;LWC of .001 kg m-3 (or 1 gm-3)
rhoH=100.                      ;density of hail (kg m-3)
dq=.1e-15                      ;0.1 fC from Avila et al (2011) JGR
                               ;This is charge per collision
mf=FLTARR(nit)
rf=FLTARR(nit)
vh=FLTARR(nit)
dQdt=FLTARR(nit)
vd=FLTARR(nit)
for t=0,nit-1 do begin

;Assume drop velocity is equal to updraft velocity and it increments
;by 1 m/s every time step
   if (t eq 0) then vd[t]=vd0 else vd[t]=vd[t-1]+1.

   if (t GT 0) then  mi=mf[t-1] ;transfer final mass to initial mass for
                                ;next timestep

;Assume fall speed of hail is twice as fast as graupel and use the
;power law relationship for fall speed from HW7 P2.  Now, V=4.8M^0.24
;Also assume vh>>vd

;Calculate final mass from previous timestep mass and is independent
;of updraft strength
   int1=alog((mi^0.52)+(0.52*23.04*!pi*wl*ec*dt))/0.52 ;kg
   int2=alog((mi^0.52)+(0.76*(2.*vd[t])*4.8*!pi*wl*ec*dt))/0.76 ;kg
   int3=mi+(!pi*(vd[t]^2.)*wl*ec*dt)

   mf[t]=int1+int2+int3        ;milligrams

;Calculate final radius from mass
   mfkg=mf[t]*1.e-6
   rf[t]=(mfkg/((4./3.)*!pi*rhoH))^(1./3.) ;meters

;calculate new velocity value of hail
   vh[t]=4.8*(mf[t])^0.24       ;m/s

;Using equation 14.2 of Lamb and Verlinde, assume epsilon is
;negligible and rd<<rh
   dQdt[t]=!pi*(rf[t]^2.)*ec*nd*nh*(vd[t]+vh[t])*dq ;C km-3 min -1

endfor

;MAKE PLOT
!p.multi=[0,2,2]
plot, vd, mf, charsize=2, $
      title='Growing Hailstone', xtitle='Updraft (m/s)', ytitle='Hail Mass (mg)'

plot, vd, rf, charsize=2, $
      title='Growing Hailstone', xtitle='Updraft (m/s)', ytitle='Hail Radius (m)'

plot, vd, vh, charsize=2, $
      title='Growing Hailstone', xtitle='Updraft (m/s)', ytitle='Hail Fall Speef (m/s)'

oplot, vd, vd, thick=2

plot, vd, dqdt, charsize=2, $
      title='Charge Rate from Induction Charging', xtitle='', ytitle='Charge Rate (C km-3 min -1)'


stop
end
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